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PHASE SPACE EFFECTS ON STICKING IN MUON CATALYSED d-t FUSION

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ABSTRACT

Within the framework of the sudden approximation, inclusion of proper phase space factors results in a 0.2% reduction of sticking in d-t fusion. The inclusion of phase space effects has been made possible by correctly interpreting the sticking fraction Ws as a ratio of rates rather than probabilities. Ws therefore makes better contact with the direct sticking experiments which measure the branching ratio rather than ws. Using similar simplified initial state wave functions, Ws is found to be 0.9% as compared to 1.1% for the conventionally defined sticking factor ws.

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INTRODUCTION

From a position of relative obscurity and insecurity in the sixties, muon catalysed fusion has achieved international prominence in the present decade.[1] Projecting the fascinating interplay of different forces of physics, it singles out an unique position for the second generation negative lepton - the muon. In addition to its importance for the fundamental interdisciplinary physics involved, its attractive potential as an exotic energy source, (by-passing tokomak temperatures and explosion dangers), makes muon catalysed fusion particularly interesting. Born out of a particle physics Bubble Chamber experiment, its study now involves a complex synthesis of particle, atomic, nuclear, accelerator, neutron and reactor physics.

The chief bottleneck to achieving efficient cold fusion is 'sticking' whereby the muon is coulombically bound to the charged fusion product and is thus lost to the catalytic chain. It is this crucial problem of sticking that this paper addresses. Since the d-t system is blessed with a lot of fusion advantages, current experiments[2,3] and theories concentrate on this combination of fusion fuel.

Existing theoretical descriptions of the sticking have recently aroused some discontent[4,5]. The naive use of the sudden approximation to determine the muon fate seems inadequate, and the need for a proper microscopic theory of the complex catalysed fusion event is increasingly felt. One way out of the impasse is to follow the suggestions in[4] and perform detailed R-matrix computations of all the reaction channels. This is no doubt a formidable task.

Neglect of the phase space factors in the conventional sudden approximation format is yet another source of dissatisfaction, that indicates this may not be the ideal description. The present work attempts to clarify at least this part of the problem. The analysis is restricted to the case of d-t fusion specifically, although the general methods used apply to different fusing partners.

The Physics and the Methodology

One must remember that the available energy after fusion is liberated at the fusion vertex and shared by the three final state particles - the alpha, neutron and spectator muon in the case of d-t fusion. The kinematic distribution of available energy has a role to play in the determination of the eventual muon fate as stuck states can occur only when the muon and alpha overlap in the final state phase space. Naturally this is a very restricted phase space volume imposing a severe suppression on the sticking. This has been discussed from a different viewpoint by Rafelskii et al[6].

The existing theories using the sudden approximation assume the sticking probability w_s is given by the ratio of the overlap of the stuck and initial states to the total dissappearance probability of the entire ($dt\mu$). But the question arises does this take into account correctly the phase space distribution and its allied suppressions ? In the direct sticking experiments one measures the branching ratio for stuck and total final states, and this in conventional particle physics format defines a ratio of the respective rates rather than probabilities. It is well known that final state kinematics and phase space effects influence reaction rates.

This work investigates the effects of proper phase space considerations on the sticking effect. The value obtained for the sticking factor is •2% less than the usual sudden approximation prediction. We borrow from the sudden approximation the philosophy that the fusion Event is so fast that the muon suddenly changes its state from the initial to final one. The matrix element is then taken as the overlap between the initial and final states for the different stuck and non-stuck cases. For the present, the fusion part of the matrix element is factored out assuming as per custom that the fate of the spectator muon is independent of the details of the fusion vertex.

The advantage of this formalism is that since the overlap integrals are interpreted as the muon part of the matrix element, one is able to introduce abinitio phase space factors and integrate over phase space. Presumably the muon is connected to the fusion vertex by vitual photon exchange which might effect the transition of the muon from initial to final state. However the two bound state wave functions include the exchange of many Coulomb photons through the usual modification of ladder diagrams. Thus it may not be necessary to include an additional virtual photon to effect the transfer. This angle will be explored in a subsequent work. In the present work it is assumed the matrix element determining the muon fate is the quantum mechanical overlap probability without invoking explictly a photon to cause the change of state. This is analogous to the treatment of recoil effects of spectator nuclei in muon decay from bound states[7]. The muon which is spectator to the actual fusion event participates in the reaction through the conservation kinematics.

The Formalism

The matrix elements for the stuck and free cases can be written

$$|M_s| = \int \Psi_i(r)\, \Phi_n(r)\, \exp(-i\underline{k}\cdot\underline{r})\, d^3\underline{r} \quad \ldots\ldots(1)$$

and

$$|M_f| = \int \Psi_i(r)\, \exp(-i\underline{p}\cdot\underline{r})\, d^3\underline{r} \quad \ldots\ldots(2)$$

$\underline{r}$ refers to the muon co-ordinate in either case, and $\underline{p}$ and $\underline{k}$ refer to the muon momentum for the free and stuck final states respectively. $\Psi_i(r)$ is the initial (dtμ) wave function taken for the present at nuclear contact, and $\Phi_n(r)$ is the final stuck muon state.

The respective rates for the two kinds of final states are then

$$\Gamma_s = (1/(2\pi)^6)\int |M_s|^2 d^3\underline{p}_{\alpha\mu}\, d^3\underline{p}_n^{\,\prime}\, (2\pi)^4 \delta^4(P_i - P_{\alpha\mu} - P_n^{\,\prime}) \quad -(3)$$

$$\Gamma_f = (1/(2\pi)^9)\int |M_f|^2 d^3\underline{p}_{\alpha}\, d^3\underline{p}_n\, d^3\underline{p}\, (2\pi)^4 \delta^4(P_i - P_{\alpha} - P - P_n) \quad -(4)$$

$\underline{p}_{\alpha\mu}$, $\underline{p}_n^{\,\prime}$ are momenta of the $(\alpha\mu)$ and the neutron respectively and $P_{\alpha\mu}$, $P_n^{\prime}$ their corresponding four momenta for the case with sticking.

$\underline{p}_{\alpha}$, $\underline{p}_n$, $\underline{p}$ are the momenta of the alpha, neutron and muon respectively for the free case without sticking, and P_{α}, P_n, P are the corresponding four-momenta.

The sticking fraction, or branching ratio for sticking W_s can now be defined as a ratio of the rates as

$$W_s = \Gamma_s / (\Gamma_f + \textstyle\sum_s \Gamma_s) \quad \ldots \quad (5)$$

where $\sum_s \Gamma_s$ sums over rates for all possible stuck states.

The value thus obtained for sticking will be compared with that obtained for the conventional sticking fraction w_s which is defined as a ratio of the probability of sticking to the total fusion probability i.e.

$$w_s = [\,|\int \Psi_i(r)\Phi_n(r) \exp(-i\underline{k}.\underline{r})\, d^3\underline{r}\,|]^2 \,/\, [\int |\Psi_i(r)|^2\, d^3\underline{r}] \ldots (6)$$

The matrix elements in both (1) and (2) are dependent on the muon momentum, after integration over configuration space. For the phase space integration in Γ_f, $|M_f|^2$ must be retained within the integrand since the muon momentum must eventually be integrated over.

However for the case of Γ_s the two particle final state is completely determined by Conservation laws and the complete phase space integration can be carried out with the help of the four-momentum δ function.

We discuss first the evaluation of Γ_s.

$$\Gamma_s = 1/(2\pi)^2 \int |Ms|^2 d^3 \underline{p}_{\alpha\mu} d^3 \underline{p}_n \delta^0(E_i - E_{\alpha\mu} - E_n)\, \delta^3(\underline{p}_i - \underline{p}_{\alpha\mu} - \underline{p}_n) \quad \ldots \quad (7)$$

The momentum δ function is used to integrate $\underline{p}_n$, and working in the rest frame of the decaying $(dt\mu)$, one gets after integrating over $\underline{p}_{\alpha\mu}$ with the energy delta function,

$$\Gamma_s = \left[|Ms|^2 E(E_i - E)\sqrt{E^2 - m^2} \right] / (\pi E_i) \quad \ldots \quad (8)$$

where $m_{\alpha\mu}$ is the mass of the $(\alpha\mu)$ ion, E_i is the total available energy and E is the energy of the $(\alpha\mu)$ as fixed by the delta function conservation.

$$E = E_i/2 + (m^2_{\alpha\mu} - m^2_n)/(2Ei) \quad \ldots \quad (9)$$

where m_n is the neutron mass and

$$Ei = \sum_i m_i = m_{\alpha\mu} + m_n + E_f - E_{dt\mu} + E_{\alpha\mu} \ldots \quad (10)$$

with m_i running over masses of all initial particles.

E_f, $E_{dt\mu}$ and $E_{\alpha\mu}$ are the energy released during fusion, the $(dt\mu)$ binding energy and the $(\alpha\mu)$ binding energy respectively.

The value of $|\underline{k}|$ in $|M_S|^2$ is fixed by the conservation laws implied in the delta functions used for the integration.

The evaluation of Γ_f is naturally slightly more complicated as it involves a 3-particle final state. This integral is evaluated in a manner analogous to free muon decay sine muon decay involves also a three particle final state. In the fusion case the (α-n) system is integrated in their centre of mass frame using the energy-momentum delta function, as for the two neutrinos in muon decay. Subsequently we move to the rest frame of the decaying initial ($dt\mu$) with $\underline{q} = \underline{p}_\alpha + \underline{p}_n$ being the total momentum carried by the α - n system.

$\underline{q}$ is given by $\underline{q} = -\underline{p}$ where $\underline{p}$ is the muon momentum

The integration over the α-n system is performed similarly to that for the($\alpha\mu$-n) system in the case of Γ_s .

However in the Γ_f case, the total energy available to the (α - n) system is (E_i $-E_\mu$) where E_μ is the energy carried by the muon. The total available energy also differs slightly from the case of Γ_s as now although E_i is still $\sum_i m_i$, the final states being now different, in the post fusion scene

$$E_i = m_\alpha + m_n + m_\mu + E_f - E_{dt\mu} \cdots \quad (11)$$

However this difference in E_i is negligible since both $|E_{dt\mu}|$ and $|E_{\alpha\mu}|$ are of order 2 keV as compared to the value of 1.8×10^4 keV for E_f . The muon energy can take values ranging from zero (when the α and n are emitted in opposite directions), to $E_i/2$ (when the muon is emitted opposite to the common direction for the α-n system.

So integrating over the (α-n) system, expression (4) for Γ_f reduces to

$$\Gamma_f = (1/(2\pi)^5)\int |M_f|^2 d^3p \left[E_\alpha(E_\mu - E_\alpha)/E_\mu\right]\sqrt{E_\alpha^2 - m_\alpha^2} \ \cdots \quad (12)$$

where $\epsilon_\mu = E_i - E_\mu$ .... (13) and E_α is again given by the delta function constraints used in the integration.

$$E_\alpha = \epsilon_\mu/2 + (m_\alpha^2 - m_n^2)/2\,\epsilon_\mu \quad \ldots \quad (14)$$

The integrand in (12) is a complicated but regular function of E_μ and can easily be integrated numerically. However as we wish to obtain an analytic expression to help in the comparitive analysis, we now make a non-relativistic approximation for the muon energy only. Thus we substitute

$$E_\mu = m_\mu + T \quad \ldots \quad (15)$$

into (12) where T is now the kinetic energy. ϵ_μ then simplifies to

$$\epsilon_\mu = M(1 - T/M) \quad \ldots \quad (16)$$

where $M = m_\alpha + m_n + E_f - E_{dt\mu}$ ... (17)

We now simplify the integrand in (12) neglecting terms $\sim T^2$ as the maximum value of T is $(E_f - E_{dt\mu})/2$ i.e. ~ 8.6MeV as compared to $M \sim 5 \times 10^3$ MeV and $m_\mu \sim 100$ MeV.

Changing over to the energy variable and using (15), (12) reduces to

$$\Gamma_f = (\sqrt{2m_\mu}/(2\pi)^5)\int_0^{E_i/2} |M_f|^2 (m_\mu + T)\sqrt{T}\, dT\, F(T) \ldots (18)$$

where $F(T) = [E_\alpha(\epsilon_\mu - E_\alpha)/\epsilon_\mu]\sqrt{E_\alpha^2 - m_\alpha^2}$ ... (19)

In evaluating F(T) we use binomial expansions for ϵ_μ and $1/\epsilon_\mu$, retaining only terms $\sim (T/M)$. Thus $(T/M)^2$ terms are neglected and these are $\sim 10^{-4}$ compared to the leading term.

Numerical Results

To compute M_s and M_f, we use a simplified wavefunction for the initial $(dt\mu)$ system as our main motivation is to compare W_s with W_s using similar wave functions.

So we use an analytic He - like wave function for the initial $(dt\mu)$ system at the point of contact of its nuclei. So

$$\Psi_i(r) = \sqrt{A^3/\pi}\ \exp(-Ar) \quad \dots \quad (20)$$

with

$$A = Z\, m_\mu\ (md + m_t)/\left\{137\ (m_d + m_t + m_\mu)\right\} \quad \dots \quad (21)$$

where $_d$, m_t are masses of the deuteron and triton respectively and Z is the effective charge of the nuclei.

We have used relativistic unit $\hbar = c = 1$ and $(e^2/hc) = \alpha = 1/137$ since we have done the phase space integrations involving the delta functions relativistically.

We take the muon mass to be unity for the energy units.

As we consider sticking into (1S) state only at first, we take

$$\Phi n\ (r) = \Phi_{1s}(r) = \exp(-A_\alpha r)\sqrt{A_\alpha{}^3/\pi} \dots \quad (22)$$

with

$$A_\alpha = Z_\alpha\ m_\alpha\ m_\mu / \left\{137\ (m_\alpha + m_\mu)\right\} \quad \dots \quad (23)$$

where Z_α is the charge of the alpha.

The configuration space integrals, are elementary and yield

$$|M_s|^2 = 64\ A^3 A_\alpha{}^3 (A+A_\alpha)^2 / \left[K^2 + (A+A_\alpha)^2\right]^4 \dots \quad (24)$$

and

$$\left| M_f \right|^2 = 64\pi A^3 A^2 / \left[p^2 + A^2 \right]^4 \ldots \quad (25)$$

Expression for Γ_f then becomes

$$\Gamma_f = G_f \int_0^{t_i/2} \sqrt{T}\,(m_\mu + T)\,d\tau\, F(T)/(A^2 + 2m_\mu T)^4 \quad \ldots \quad (26)$$

the two integrals over T are of type $\int \frac{x\sqrt{x}\,dx}{(a+bx)^4}$ and $\sqrt{x}dx/(a+bx)^4$ and can be done analytically. (G_f is constant).

Substituting all necessary values for masses and energies one gets finally

$\Gamma_f = 4.39$ and $\Gamma_s = 0.039$

so that

$W_s = \Gamma_s / (\Gamma_f + \Gamma_s) = 0.0088$

or $W_s = .9$ %.

(In the denominator, $\sum_{s'} \Gamma_{s'}$ where S' refers to stuck states other than IS are ignored for the present, and would further reduce W_s . Since sticking into IS is known to be largest, additional terms in the denominator less than $\sim$ .01 can be neglected in this estimate.

Since the initial wave function is normalised, the expression (6) for ω_s becomes identical to $\left| M_s \right|^2$ and numerically has a value

w_s = 0.011 corresponding to a sticking fraction of 1.1%

Thus the value obtained for sticking including phase space effects is .9% as compared to 1.1% without these.

Discussion

It is extremely encouraging that proper inclusion of phase space effects reduces the theoretical value of the branching ratio for sticking. This brightens of course prospects of eventual utilisation.

It is known that use of exact variational wave-functions, taken at contact yield a lower value for w_s as defined by expression (6) than the 1.1% obtained here. However the phase space reduction of the effective branching ratio will still be valid so that use of a similar exact wave function in the expression for W_s will cause W_s to be also correspondingly lower than .9% obtained here.

This work demonstrates that under identical initial conditions, and factoring out the muon line in the catalysed fusion matrix element, correct computation of the sticking fraction as a branching ratio gives it a value (.2%) lower than the interpretation of w_s as a probability without inclusion of phase space effects.

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